Article

Quasi-classical model for real space shape of the Cooper pair probability density

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Within Bardeen-Cooper-Schrieffer theory of superconductivity, two electrons form the Cooper pair in the momentum space. However, it is a challenging task to represent the Cooper pair probability density in real space. Here we proposed a quasi-classical three-dimensional model for the Cooper pair probability density shape in a real space, which is appeared as a direct consequence to describe the Meissner-Ochsenfeld critical field, $B_{c,MO}$ (which is the thermodynamic critical field in Type-I superconductors, and lower critical field in Type-II superconductors) by the equation $B_{c,MO} = 1/2\mu_0 n\mu_B ln(1+\sqrt{2}\xi/\lambda)$, where μ_0 is the magnetic constant of SI system, n is the Cooper pair bulk density, μ_B is the Bohr magneton, and λ is the London penetration depth, and ξ is the coherence length. As a result, the Meissner-Ochsenfeld field can be defined as the field at which each Cooper pair exhibits the diamagnetic moment of one Bohr magneton with a multiplicative pre-factor of $1/2 \cdot ln(1+\sqrt{2} \cdot \lambda/\xi)$. Based on quasi-classical interpretation of this result, in this study we proposed that the probability density of the Cooper pair in real space can be represented as a toroid with an inner radius ξ and an outer radius of $\xi + +\sqrt{2}\lambda$. This means that the superconducting transition is associated not only with the charge carrier pairing, but that the pairs exhibit a new topological state with genus 1.

Keywords: cooper pair; topological phase transition; coherence length; London penetration depth.

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1. Introduction

Walther Meissner and Robert Ochsenfeld discovered that an external magnetic field, B, is expelled from superconducting tin and lead [1]. Each superconductor exhibits the maximum magnetic flux density, B_{MO}, at which the superconducting state starts to collapse. This field can be described by the following equations [2]:

$$B_c(T) = \frac{\phi_0}{2\sqrt{2}\pi}\frac{1}{\lambda(T)\xi(T)}, \quad \text{for Type-I superconductors} \tag{1}$$

$$B_{c1}(T) = \frac{\phi_0}{4\cdot\pi}\frac{ln\left(\frac{\lambda(T)}{\xi(T)}\right)+0.5}{\lambda^2(T)}, \quad \text{for Type-II superconductors} \tag{2}$$

where λ is the London penetration depth, ξ is the coherence length, $\phi_0 = h/(2e) \approx 2.07 \times 10^{-15}$ is the superconducting flux quantum, where h is the Planck constant, and e is the electron charge. The effect of magnetic flux expelling from superconductors was recognized as one of the most fundamental effects in superconductivity [3-11], has and this effect has been utilized in several superconducting technologies [12-17].

It should be stressed that if for Type-II superconductors the appearance of the superconducting flux quantum, ϕ_0, in the Eq. 2 can be justified by the presence of Abrikosov vortices in the superconductor where each vortex holds ϕ_0 flux, the presence of ϕ_0 in Eq. 1 cannot be unjustified, because Type-I superconductors do not form vortices and flux quantation concept cannot be applicable for these superconductors.

However, both types of superconductors exhibit the same fundamental particle, i.e. the Cooper pair, which is formed by two electrons or holes in the momentum space [5]. It should be mentioned, that Bardeen, Cooper and Schrieffer [5] did not specify the geometry or spatial confinement for the Cooper pairs in the real space. Hirsch [6] proposed a geometrical real space model (by assuming that classical mechanics and classical electrodynamics) for electrons motion in a Cooper pair. In this model, two charge carriers circulate in opposite directions in orbits of radius $R=2\lambda$ with the centers of their orbits separated by ξ.

Here we point out, that if both equations 1, 2 can be represented by the universal equation [18,19]:

$$B_{MO} = \frac{\phi_0}{4\pi} \frac{\ln(1+\sqrt{2}\frac{\lambda}{\xi})}{\lambda^2}, \tag{3}$$

where subscript MO designates the Meissner-Ochsenfeld field (i.e. the flux density at which the superconducting state starts to collapse), it is more natural to use different real space arrangement for two electrons motion in the Cooper pair. This arrangement is a toroid with an inner radius ξ and an outer radius of $\xi+\sqrt{2}\lambda$. This implies that at superconducting transition a new topological state of the matter arises. This state is characterized by the charge carriers which have double charge and real space probability density in form of torus. Thus, our primary conclusion is that the charge carriers in the superconducting state exhibits a new topological state with genus 1.

Here we presented quasi-classical derivation of this statement, where classical mechanics is used to describe the charge carriers motion and new particle space confinement. After that, the interpretation of obtained results will be presented in the form of probability density.

2. Bohr magnetons density in a superconductor

In type-II superconductors, for which the Ginzburg-Landau parameter $\kappa(T)\geq 1/\sqrt{2}$, the Meissner-Ochsenfeld field is called the lower critical field, $B_{c1}(T)$, for which Brandt proposed more accurate expression, than Eq. 2 [20]:

$$B_{c1}(T) = \frac{\phi_0}{4\pi} \frac{ln(\kappa(T)) + \alpha(\kappa(T))}{\lambda^2(T)}, \tag{4}$$

where

$$\alpha(k(T)) = 0.49693 + e^{(-0.41477-0.775ln(k(T))-0.1303(ln(k(T)))^2)}. \tag{5}$$

However, Eqs. 4,5 are too complicated in any practical use and recently Eqs. 1, 2, 4, 5 were combined in one universal equation [18,19]:

$$B_{c,MO}(T) = \frac{\phi_0}{4\pi} \frac{ln\left(1+\sqrt{2}\kappa(T)\right)}{\lambda^2(T)}. \tag{6}$$

Eq. 6 still contains ϕ_0 and, thus, there is a remaining problem with physical interpretation of Eq. 6 for type-I superconductors, where vortices do not exist. Based on this, we further simplify the Eq. 6 to reveal the background physics behind this.

Eq. 6 can be re-written in the form:

$$B_{c,MO}(T)=\mu_0\left(\frac{\hbar e}{2m_e}\right)\left(\frac{1}{4\mu_0 e^2}\frac{m_e}{\lambda^2(T)}\right)ln\left(1+\sqrt{2}\kappa(T)\right)=\frac{1}{2}\mu_0\mu_B n(T)ln\left(1+\sqrt{2}\kappa(T)\right), \quad (7)$$

where $\mu_B=\hbar e/(2m_e)$ is the Bohr magneton, and n(T) is the bulk density of Cooper pairs in the material which is given by:

$$n(T)=\frac{1}{2}\frac{1}{\mu_0 e^2}\frac{m_e}{\lambda^2(T)}. \quad (8)$$

Eq. 7 can be interpreted to say that maximum diamagnetic response in the superconductor is achieved when each Cooper pair in the material exhibits a magnetic momentum of one Bohr magneton with a multiplicative pre-factor of 1/2ln(1+√2κ(T)). It is important to note that the Bohr magneton has the same value for single and double charges:

$$\mu_B=\frac{\hbar e}{2m_e}=\frac{\hbar(2e)}{2(2m_e)} \quad (9)$$

This means that the magnetic moment of the Cooper pair:

$$\mu_{Cooper\ pair}(T)=\frac{1}{2}\mu_B ln\left(1+\sqrt{2}\frac{\lambda(T)}{\xi(T)}\right)=\frac{1}{2}\mu_B ln\left(\frac{\xi(T)+\sqrt{2}\lambda(T)}{\xi(T)}\right) \quad (10)$$

requires a different physical interpretation, which we presented in the next Section, where we utilized quasi-classical consideration for the two-dimensional (2D) model (because the simplicity of this model).

3. Ring magneton

Consider a model, where a thin disk with a large concentric hole lies in the xy plane, centered on the origin (Fig. 1). The disk has an inner radius, a, and an outer radius, b.

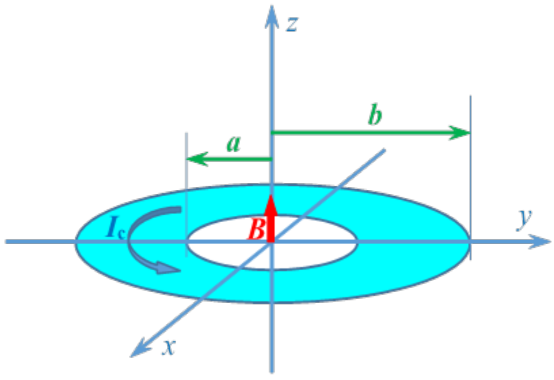

Fig. 1. Schematic diagram of the model. The vector of magnetic flux density in the center of the disk, B, is shown.

The disk carries a uniformly distributed surface current, I_c, by which we understand that the charge is distributed uniformly within the disc and the tangential charge velocity, v_0, is constant within the disc (Fig. 2 (a)). This means that each elemental concentric coil of radius r and width dr is carrying the same current, dI, independent of r (this is because the electric current is defined as the rate of electric charge flowing through the cross-section of the conductor (which in our 2D model is dr)). It needs to be clarified that this current is created by the movement of the central mass of a Cooper pair.

In our model (Fig. 2(a)), each thin disk with radius r and width dr creates a magnetic flux density, dB, in the center of the disc:

$$dB = \frac{\mu_0}{2} I_c \frac{dr}{b-a} \frac{1}{r} \tag{11}$$

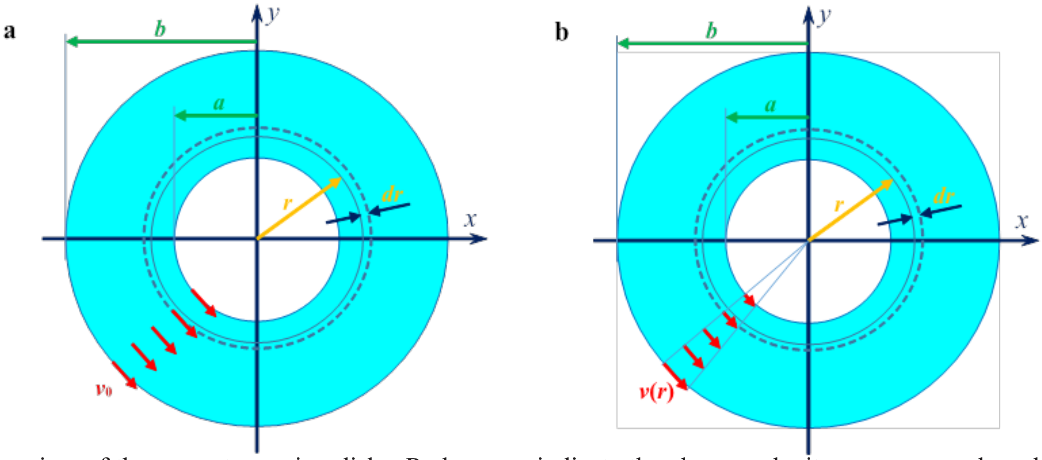

Fig. 2. Plan view of the current-carrying disks. Red arrows indicate the charge velocity. a – proposed model; b – solid magneton model.

Integration of Eq. 11 gives the total magnetic flux density in the center of the disk:

$$\int_a^b dB = \frac{\mu_0}{2} \frac{I_c}{b-a} \int_a^b \frac{dr}{r} = \frac{\mu_0}{2} \frac{I_c}{b-a} ln\left(\frac{b}{a}\right) \tag{12}$$

By comparing Eq. 12 with Eq. 10 it is natural to propose that:

$$\begin{cases} a = \xi \\ b = \xi + \sqrt{2}\lambda \end{cases} \tag{13}$$

In this case, Eq. 12 can be written in the form:

$$B_{center} = \mu_0 \frac{I_c}{\sqrt{2}\lambda} \frac{1}{2} ln\left(1 + \sqrt{2}\frac{\lambda}{\xi}\right) \tag{14}$$

Where the logarithmic multiplicative term of $1/2\ ln(1+\sqrt{2}\ \lambda/\xi)$ is the same as that in Eq. 10. The natural appearance of $1/2\ ln(1+\sqrt{2}\ \lambda/\xi)$ in Eqs. 12-14 is a direct consequence of the model, which postulates that the ring magneton has constant current, $I_c(r)$ (Fig. 2,a), independent of its radius.

Within this model, in low-k superconductors the disk is a very thin hoop, while for high-k superconductors the disk is a large ring with a tiny central hole. It should be noted that due to the ring geometry, the magnetic flux density, B, is concentrated inside the inner diameter of the ring.

The alternative model of a solid magneton can also be considered (Fig. 2,b). In this case, the tangential charge velocity, v(r) is a linear function of r (Fig. 2,b). Thus, to keep the electric current, dI, carried by the elemental concentric coil with radius r and width dr constant, the surface charge carrier density should be a reciprocal function of r. This model also gives the multiplicative pre-factor of $1/2 ln(1+\sqrt{2}\ \lambda/\xi)$ for the Bohr magneton $\hbar e/(2m_e)$ value. However, there is a very important feature of both models, which is the ring spatial confinement of the Cooper pair.

Because the two charge carriers of the Cooper pairs are moving in opposite directions, the net current within a ring is zero if an external field is not applied. However, when a field is applied, then the centre of mass of the Cooper pairs precesses and this creates the diamagnetic moment. Thus, we propose that the essential condition at which the multiplicative pre-factor of $ln(1+\sqrt{2}\ \lambda/\xi)$ appears in Eqs. 6, 7, 10 is a new topological state for paired charge carriers (i.e., a ring in 2D case).

3.1 Ring toss model for magnetic flux tube in superconductor

If external magnetic field is applied at the sample surface, B_{appl}, and this field exceeds $B_{c,MO}(T)$, than the magnetic flux penetrates into the superconductor in the form of magnetic flux tubes. In our model, because the Cooper pair center of mass does not occupy a space inside the ring, it is natural to expect that the magnetic flux tube will be trapped there, i.e. magnetic flux will be concentrated inside of a ring within a circle with radius ξ. As far as Cooper pairs rings will be rearranged within the whole sample volume, then the flux tube will appear in the usual manner of a vortex line. Our interpretation of flux lines is similar, but not exactly identical, to the concept of the Abrikosov vortex [21]. The difference is that in the Abrikosov vortex model, the superconducting state inside of the core of radius ξ is completely suppressed. However, the flux density which can suppress the superconducting state should be about the Pauli depairing field, $B_p(T)$:

$$B_p(T) = \frac{2\Delta(T)}{g\mu_B} \gg 4B_{c1}(T) = 4B_{c,MO}, \tag{15}$$

where Δ(T) is the superconducting energy gap, and g = 2, which is at least one order of magnitude greater than the field at the radius R = ξ for the core of the single Abrikosov vortex (see, for instance, Eq. 12.18 in [22]):

$$B(r=\xi) = B_{c1}\frac{K_0\left(\frac{\xi}{\lambda}\right)}{\frac{1}{2}ln\left(\frac{\lambda}{\xi}\right)} \approx 2B_{c1}\frac{2ln\left(\frac{\lambda}{\xi}\right)-0.6}{ln\left(\frac{\lambda}{\xi}\right)} \lesssim 4B_{c1} = 4B_{c,MO}, \tag{16}$$

where K_0 (x) is a zeroth-order modified Bessel function.

In our model, if the applied field B_{appl} is not so high that the single vortex model can be a good approximation, the superconducting state inside of the flux tube is not destroyed by the field, but instead the superconducting state does not exist inside of the tube because of Cooper pair ring spatial confinement. The flux tube, thus, holds surrounding tossed rings of Cooper pairs. It should also be mentioned that Cooper pairs can persistently surround columnar non-superconducting nanowires embedded in the superconducting matrix [23-28].

3.2. Critical current density

Taking into account that:

$$B_{c,MO}(T) = \mu_0 J_c(T)\lambda(T), \tag{17}$$

where $J_c(T)$ is the maximum dissipation-free current density in the surface layer of the superconductor, and combining with Eq. 17 with Eq. 8 one can derive the equation:

$$J_c(T) = \frac{n(T)}{\lambda(T)}\left[\mu_B ln\left(1+\sqrt{2}\frac{\lambda(T)}{\xi(T)}\right)\right] = \sqrt{\frac{2}{\mu_0 e^2 m_e}}\left[\mu_B ln\left(1+\sqrt{2}\frac{\lambda(T)}{\xi(T)}\right)\right]n^{1.5}(T) \tag{18}$$

Eq. 18 shows that the critical current density in the material is proportional to the density of Cooper pairs at the power of 1.5. It can be shown that the low-temperature low-k limit of Eq. 18 has a pair-breaking form:

$$J_c(0) \sim n(0)\Delta(0), \tag{19}$$

where Δ(0) is the ground state energy gap. Thus,

$$J_c(T \to 0) = \frac{n(T \to 0)}{\lambda(T \to 0)}\left[\mu_B ln\left(1+\sqrt{2}\frac{\lambda(T \to 0)}{\xi(T \to 0)}\right)\right] \cong \left[\frac{\pi e}{\sqrt{2m_e}}\right]\frac{1}{v_F} n(0)\Delta(0), \tag{20}$$

where, $\xi(0)=\hbar v_F/\pi\Delta(0)$ and ν_F is the Fermi velocity in the material. Despite that for low-k materials J_c can be expressed in the form of pair-breaking concept (Eqs. 19,20), the dependence of J_c on the density of Cooper pairs to the power of 1.5 is universal for both low-k and high-k materials.

4. Discussion

It should be noted that neither Cooper [29] nor Bardeen, Cooper and Schrieffer [5] specified the real space spatial confinement or probability density for the Cooper pairs. Hirsch [6] proposed a geometrical quasi-classical model for a Cooper pair, where two charge carriers circulate in opposite directions in orbits of radius 2·λ with the centers of their orbits separated by ξ. However, this model cannot give the multiplicative pre-factor of *ln(1+√2·λ/ξ)* to the Bohr magneton, until the idea of the spatial confinement of the center of mass of the charge carriers within a disc with inner radius ξ and outer radius of *ξ+√2λ* is implemented.

We need to stress that a change in the system topology at a phase transition is a common feature which has been first proposed by Berezinsky [30,31], and Kosterlitz and Thouless [32]. Taking this into account, our finding that the superconducting transition is not only associated with the creation of Cooper pairs, but it also associated with the enriching system topology by new objects with genus 1, agrees with the Berezinsky-Kosterlitz-Thouless [30-32] physical law.

It is interesting to note that our ring model , and its interpretation as the probability density of the Cooper pair in real space, provides a simple geometrical justification for the division between low- k and high- k materials. This boundary is set at

$$\kappa = \left(\frac{1}{2}\right)^{\frac{1}{2}} \tag{21}$$

Thus, at this k value the ring width, w, is equal to the inner radius:

$$w = \left(\xi + \sqrt{2}\lambda\right) - \xi = \sqrt{2}\kappa\xi = \sqrt{2}\frac{1}{\sqrt{2}}\xi = \xi \tag{22}$$

This means that the ring in low-k materials (Type-I) satisfies the geometrical confinement:

$$w \le \xi \tag{23}$$

while in high- k (Type-II) materials:

$$\xi \le w. \tag{24}$$

A schematic representation of the ring with κ=(1/2)^(1/2) for which the outer radius:

$$(\xi + \sqrt{2}\lambda) = \xi + (2)^{\frac{1}{2}}(\frac{1}{2})^{\frac{1}{2}}\xi = 2\xi \tag{25}$$

and the inner radius is ξ is shown in Fig. 3.

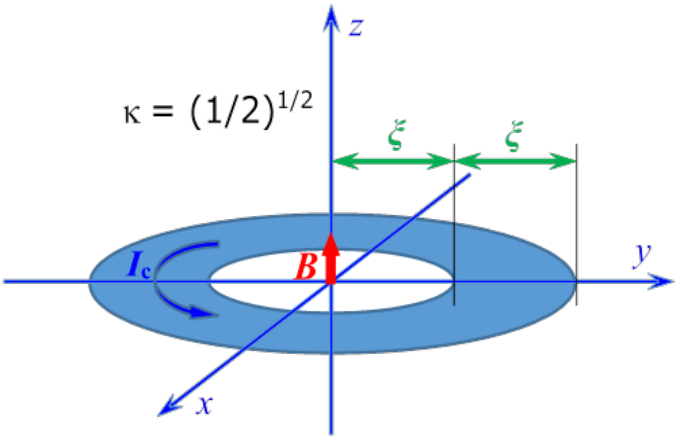

Fig. 3. Schematic representation of the Cooper pair ring for κ=(1/2)^(1/2). The I_c represents the net current originating from the circulation of the Cooper pair center of mass.

Returning now to the global response of the superconducting sample to an applied magnetic field, B_{appl}, we should mention that Condon [33] had proposed an empirical expression for the ground state $B_{c,MO}(0)$ which covers elemental metallic superconductors (i.e., Pb, Ta, Sn, Hg, In, Tl, Nb, and Zr which all are low-k materials) known at the present time:

$$B_{c,MO}(0) = \mu_0 \mu_B \times \delta \times \frac{1}{5N}, \tag{26}$$

where δ is the number of atoms per unit volume, and N =1/2, 1, 2, 3, 4. A comparison of Eq. 7 with Eq. 26 shows that:

$$\delta \frac{1}{5N} = \frac{1}{2} \times n \times ln\left(1+\sqrt{2}\kappa\right), \tag{27}$$

where δ and n have similar physical meaning for the density of elemental diamagnets per unit volume, and multiplicative pre-factors of *1/5N* and *ln(1+√2 κ)* represent the "strength" of the Cooper pair ring magnetic moment in comparison with the magnetic moment of classical Bohr magneton.

We need to stress that presented quasi-classical model utilizes classical mechanics and classical electrodynamics to find spatial confinement of the Cooper pairs in type-I and type-II superconductors, and when this confinement has found, we interpreted the obtained result in terms of probability density (the latter is the fundamental approach utilized in quantum physics).

It should be mentioned that within our model, the Cooper pair is considered as a new particle, which has the electric charge of 2e and temperature dependent magnetic momentum

$$\mu_{Cooper\ pair}(T) = \frac{1}{2}\mu_B ln\left(\frac{\xi(T)+\sqrt{2}\lambda(T)}{\xi(T)}\right), \tag{28}$$

while Hirsch in his model [6] considered the Cooper pair as an object which contains two separate electrons.

5. Summary

In this paper we specified the topology for Cooper pairs probability density in superconductors. As a result of our analysis, we proposed a probability density for the Cooper pair having shape of the ring, which is a new topological state in the superconducting state of the conductor. The spatial shape has the center of mass within the inner radius ξ and outer radius $\xi+\sqrt{2}\times\lambda$. This means that the superconducting transition is associated with the change in system topology by the appearance of the Cooper pairs which have probability density in real space in form of toroid, which exhibit a topological state of genus 1. A universal equation for the Meissner-Ochsenfeld critical field, $B_{c,MO}$, which is based on the concept of the Cooper pair toroid and which covers both low-k and high-k superconductors, has been proposed

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